

Detection of small exchange fields in S/F structures

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Abstract

Ferromagnetic materials with exchange fields E_{ex} smaller or of the order of the superconducting gap Δ are important for applications of corresponding (s-wave) superconductor/ ferromagnet/ superconductor (SFS) junctions. Presently such materials are not known but there are several proposals how to create them. Small exchange fields are in principle difficult to detect. Based on our results we propose reliable detection methods of such small E_{ex} . For exchange fields smaller than the superconducting gap the subgap differential conductance of the normal metal - ferromagnet - insulator - superconductor (NFIS) junction shows a peak at the voltage bias equal to the exchange field of the ferromagnetic layer, $eV = E_{\text{ex}}$. Thus measuring the subgap conductance one can reliably determine small $E_{\text{ex}} < \Delta$. In the opposite case $E_{\text{ex}} > \Delta$ one can determine the exchange field in scanning tunneling microscopy (STM) experiment. The density of states of the FS bilayer measured at the outer border of the ferromagnet shows a peak at the energy equal to the exchange field, $E = E_{\text{ex}}$. This peak can be only visible for small enough exchange fields of the order of few Δ .

Key words: exchange field, S/F hybrid structures, proximity effect

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1. Introduction

As we know from the quantum theory of magnetism the ferromagnetic metal can be described by the presence of the so called exchange field, E_{ex} . This field is responsible to many interesting phenomena in artificially fabricated superconductor/ ferromagnet (S/F) hy-

brid structures [1–4]. Let us briefly review the essence of the S/F proximity effect.

Upon entering of the Cooper pair into the ferromagnetic metal it becomes an evanescent state and the spin up electron in the pair lowers its potential energy by E_{ex} , while the spin down electron raises its potential energy by the same amount. In order for each electron to conserve its total energy, the spin up electron must increase its kinetic energy, while the spin down electron must decrease its kinetic energy, to make up for these additional potential energies in F. As a consequence, the cen-

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ter of mass motion is modulated and superconducting correlations in the F layer have the damped oscillatory behavior [5,6]. If we neglect the influence of other possible parameters of ferromagnetic metal (like magnetic scattering rate, etc.) the characteristic lengths of the decay and the oscillations are equal to $\xi_f = \sqrt{\mathcal{D}_f/E_{\text{ex}}}$, where \mathcal{D}_f is the diffusion coefficient in the ferromagnetic metal [1].

The length ξ_f is also the length of decay and oscillations of the critical current in Josephson S/F/S junctions [7,8]. Negative sign of the critical current corresponds to the so-called π -state [9–13]. S/F/S π -junctions have been proposed as potential elements in superconducting classical and quantum logic circuits [14–16]. For instance, S/F/S junctions can be used as complementary elements (π -shifters) in RSFQ circuits (see Ref. [17] and references therein). S/F/S based devices were also proposed as elements for superconducting spintronics [18]. Finally, S/F/S structures have been proposed for the realization of so called φ -junctions with a φ drop in the ground state, where $0 < \varphi < \pi$ [19,20].

Presently known ferromagnetic materials have large exchange fields, $E_{\text{ex}} \gg \Delta$ and therefore short characteristic length of oscillations, $\xi_f \ll \xi_s$, where $\xi_s = \sqrt{\mathcal{D}_s/2\Delta}$ is the superconducting coherence length and \mathcal{D}_s is the diffusion coefficient in the superconductor. This requires very high precision in controlling the F layer thickness in the fabrication process of the Josephson π -junctions. In already existing S/F/S structures the roughness is often larger than the desired precision. The way to solve this problem is to invent ferromagnetic materials with small exchange fields.

In this paper we review several proposals for ferromagnetic materials with exchange fields E_{ex} smaller or of the order of the superconducting gap Δ . Then based on our results we propose reliable detection methods of such small exchange fields in experiments. Another detection method was recently suggested in [21].

2. Ways to generate small exchange fields

The easiest way to create small exchange field is to apply an external magnetic field B to the normal metal lead, in which case $E_{\text{ex}} = \mu_B B$, where μ_B is the Bohr magneton.

It may be also an intrinsic exchange field of weak ferromagnetic alloys. For example, in Ref. [22] were reported exchange fields for $\text{Pd}_{1-x}\text{Ni}_x$ with different Ni concentration, obtained by a fitting procedure (see also [23]). Considering Nb as a superconductor with $\Delta = 1.3$ meV, we can estimate the exchange field in $\text{Pd}_{1-x}\text{Ni}_x$:

for 5.5% of Ni fitting gives $E_{\text{ex}} = 0.11$ meV, which is 0.1Δ , for 6% of Ni it gives $E_{\text{ex}} = 0.45$ meV, which is 0.4Δ , for 7% of Ni it gives $E_{\text{ex}} = 2.8$ meV, which is 2.2Δ , and for 11.5% of Ni $E_{\text{ex}} = 3.9$ meV, which is 3Δ .

Another promising alloy with small exchange field, $\text{Pd}_{0.99}\text{Fe}_{0.01}$, was studied in [24–26].

Finally, a small exchange field can be induced by a ferromagnetic material into the adjacent normal metal layer. In recent proposal [27], a thin normal metal layer was placed on top of the ferromagnetic insulator. It was shown that the ferromagnetic insulator may induce effective exchange field in the normal metal layer [27],

$$E_{\text{ex}}^{\text{eff}} = \hbar \mathcal{D} G_{\phi} \rho / d, \quad (1)$$

where \mathcal{D} is the diffusion coefficient in the normal metal, G_{ϕ} is a surface conductancelike coefficient for the normal metal/ ferromagnetic insulator interface, ρ is the resistivity of the normal metal, and d is the thickness of the normal metal layer in the direction, perpendicular to the ferromagnetic insulator surface. The field $E_{\text{ex}}^{\text{eff}}$ is expected to be much smaller than the exchange field inside standard ferromagnets. Interestingly, such exchange field is possible to tune at the sample fabrication stage since it is inversely proportional to the normal metal layer thickness d . This gives a flexibility with respect to material constraints. We also note that as $E_{\text{ex}}^{\text{eff}} \propto G_{\phi}$, inducing the tunnel barrier at the normal metal/ ferromagnetic interlayer interface, one can further reduce the value of the effective exchange field.

Below we suggest direct measurements of such small exchange fields. The detection methods are different in case of the exchange field smaller, $E_{\text{ex}} < \Delta$, and larger than the superconducting gap, $E_{\text{ex}} > \Delta$.

We should mention that we propose methods of small exchange field detection in the ideal case of ferromagnetic layer with homogeneous magnetization and absence of magnetic and spin-orbit scattering in contact with a superconductor. However, in case of realistic ferromagnets situation can be more complicated. We discuss some possible limitations of the detection in the end of the two following sections.

3. Detection of exchange fields smaller than the superconducting gap

In this section we consider the following SIFN structure: a ferromagnetic wire F of a length d_f (smaller than the inelastic relaxation length [28,29]) is attached at $x = 0$ to a superconducting (S) and at $x = d_f$ to a normal (N) electrode. The interface at $x = 0$ is a tunnel barrier while at $x = d_f$ we have a transparent interface. We will show that the subgap differential conductance

of such a structure has a peak at the bias voltage equal to the exchange field of the ferromagnetic metal in case when $E_{\text{ex}} < \Delta$ [30,31]. Thus we propose to determine small $E_{\text{ex}} < \Delta$ in experiments by measuring the subgap differential conductance of NFIS junctions at low temperatures.

In this paper we consider the diffusive limit, i.e. we assume that the elastic scattering length is much smaller than the decay length of the superconducting condensate into the F region. Here and below we consider for simplicity $\mathcal{D}_f = \mathcal{D}_s \equiv \mathcal{D}$ and $\hbar = k_B = 1$. In order to describe the transport properties of the system we solve the Usadel equation in the F layer, that in the so called θ -parametrization reads [32,33]

$$\frac{\mathcal{D}}{2i} \partial_{xx}^2 \theta_{f\uparrow(\downarrow)} = (E \pm E_{\text{ex}}) \sinh \theta_{f\uparrow(\downarrow)}. \quad (2)$$

Here the positive and negative signs correspond to the spin-up \uparrow and spin-down \downarrow states, respectively. Because of the high transparency of the F/N interface the functions $\theta_{f\uparrow(\downarrow)} = 0$ at $x = d_f$. While at the tunneling interface at $x = 0$ we use the Kupriyanov-Lukichev boundary condition [34]

$$\partial_x \theta_{f\uparrow(\downarrow)}|_{x=0} = \frac{R_F}{d_f R_T} \sinh[\theta_{f\uparrow(\downarrow)}|_{x=0} - \Theta_s], \quad (3)$$

where R_F and R_T are the normal resistances of the F layer and SF interface, respectively ($R_T \gg R_F$), and $\Theta_s = \text{arctanh}(\Delta/E)$ is the superconducting bulk value of the parametrization angle in the S layer, θ_s . Once the functions $\theta_{f\uparrow(\downarrow)}$ are obtained one can compute the current through the junction. In particular we are interested in the Andreev current, i.e. the current for voltages smaller than the superconducting gap due to Andreev processes at the S/F interface.

Due to the tunneling barrier at the S/F interface the proximity effect is weak and hence we linearize Eqs. (2-3) with respect to $R_F/R_T \ll 1$. After a straightforward calculation we obtain the Andreev current at zero temperature in this limit [35,36],

$$I_A = \frac{W\Delta^2}{4eR_T} \sum_{j=\pm} \int_0^{eV} \frac{dE}{\Delta^2 - E^2} \times \text{Re} \left[\sqrt{\frac{i\Delta}{E + jE_{\text{ex}}}} \tanh \left(\sqrt{\frac{E + jE_{\text{ex}}}{i\Delta}} \frac{d_f}{\xi_s} \right) \right], \quad (4)$$

where $W = \xi_s R_F / d_f R_T$ is the diffusive tunneling parameter [34,37,38]. In the tunneling limit $W \ll 1$.

We evaluate Eq.(4) in the long-junction limit, i.e. when $d_f \gg \xi_f$, and $E_{\text{ex}} \lesssim eV < \Delta$. We obtain for the Andreev current

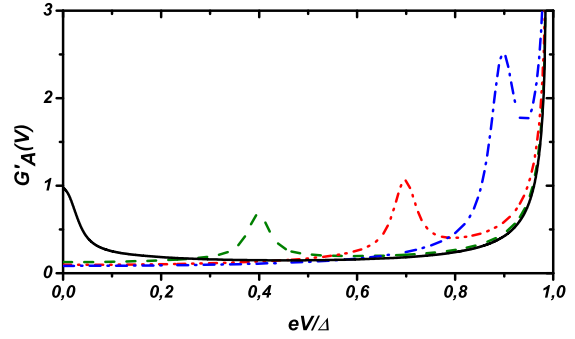


Fig. 1. (Color online) The bias voltage dependence of differential conductance at zero temperature for exchange fields, $E_{\text{ex}}/\Delta = 0$ (black solid line), $E_{\text{ex}}/\Delta = 0.4$ (green dashed line), $E_{\text{ex}}/\Delta = 0.7$ (red dash-dot-dotted line), and $E_{\text{ex}}/\Delta = 0.9$ (blue dash-dotted line). Here $G'_A = 2R_T G_A$, $W = 0.014$ and $d_f = 10\xi_s$.

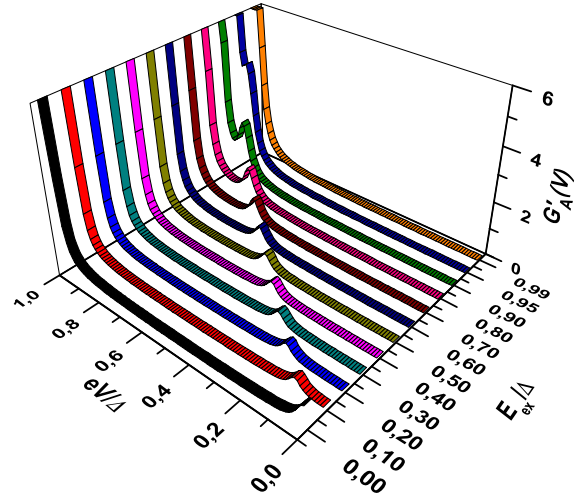


Fig. 2. (Color online) The bias voltage dependence of differential conductance at zero temperature for exchange fields, $E_{\text{ex}}/\Delta = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95$ and 0.99 . Here $G'_A = 2R_T G_A$, $W = 0.014$ and $d_f = 10\xi_s$.

$$I_A = \frac{\Delta \xi_s R_F}{e d_f R_T^2} \sum_{j=\pm} \frac{\text{arctanh}(c_j^+) + \text{arctan}(c_j^-)}{\sqrt{\Delta + jE_{\text{ex}}}}, \quad (5)$$

$$c_j^+ = \sqrt{\frac{eV + jE_{\text{ex}}}{\Delta + jE_{\text{ex}}}}, \quad c_j^- = \sqrt{\frac{eV - jE_{\text{ex}}}{\Delta + jE_{\text{ex}}}}.$$

In Fig. 1 we plot the Andreev differential conductance $G_A = dI_A/dV$ which is equal to the full differential conductance of the junction at zero temperature. The conductance shows two well defined peaks, one at $eV = E_{\text{ex}}$ and the other at $eV = \Delta$. The detailed physical explanation of the peak at E_{ex} is given in [30]. It turns out that it is the zero bias anomaly (ZBA) peak for the

diffusive NIS junction, shifted in FIS case by E_{ex} . The ZBA peak in NIS is shown in Fig. 1 by black solid line; $E_{\text{ex}} = 0$ corresponds to the normal metal case.

In Fig. 2 we plot the Andreev differential conductance for many different values of $0 \leq E_{\text{ex}} < 1$ to show the evolution of the peak with increasing E_{ex} . Detecting this peak one can carefully measure the value of small exchange field $E_{\text{ex}} < \Delta$ in the ferromagnetic metal.

We would like to mention that the peak will be visible at $eV = E_{\text{ex}}$ for a single domain ferromagnet in contact with a superconductor. In case of a multi-domain ferromagnet the peak of the differential conductance occurs at eV equals to the “effective field”, which is the field acting on the Cooper pairs in the multi-domain ferromagnetic region, averaged over the decay length of the superconducting condensate into a ferromagnet [39,40].

4. Detection of exchange fields larger than the superconducting gap

In this section we consider just a simple FS bilayer with a transparent interface: wire F of a length d_f (smaller than the inelastic relaxation length [28,29]) is attached at $x = 0$ to a superconducting electrode by a transparent interface. We will show that the density of states (DOS) measured at the outer border of the ferromagnet ($x = d_f$) shows a peak at the energy equal to the exchange field for $d_f \gg \xi_f$ in case when E_{ex} is of the order of few Δ [8,41]. Thus we propose to determine $E_{\text{ex}} > \Delta$ in experiments by measuring the DOS at the outer border of the ferromagnetic metal in corresponding SF bilayer structure, which can be done by scanning tunneling microscopy (STM).

The DOS $N_f(E)$ normalized to the DOS in the normal state, can be written as

$$N_f(E) = [N_{f\uparrow}(E) + N_{f\downarrow}(E)] / 2, \quad (6)$$

where $N_{f\uparrow(\downarrow)}(E)$ are the spin resolved DOS written in terms of spectral angle θ_f ,

$$N_{f\uparrow(\downarrow)}(E) = \text{Re} [\cosh \theta_{f\uparrow(\downarrow)}]. \quad (7)$$

To obtain N_f , we use a self-consistent two-step iterative procedure [8,42]. In the first step we calculate the pair potential coordinate dependence $\Delta(x)$ using the self-consistency equation in the S layer in the Matsubara representation. Then, using the $\Delta(x)$ dependence, we solve the Usadel equations in the S layer,

$$\frac{\mathcal{D}}{2i} \partial_{xx}^2 \theta_s = E \sinh \theta_s + i \Delta(x) \cosh \theta_s, \quad (8)$$

together with the Usadel equations in the F layer [Eq. (2)] and corresponding boundary conditions, repeating the iterations until convergence is reached [8].

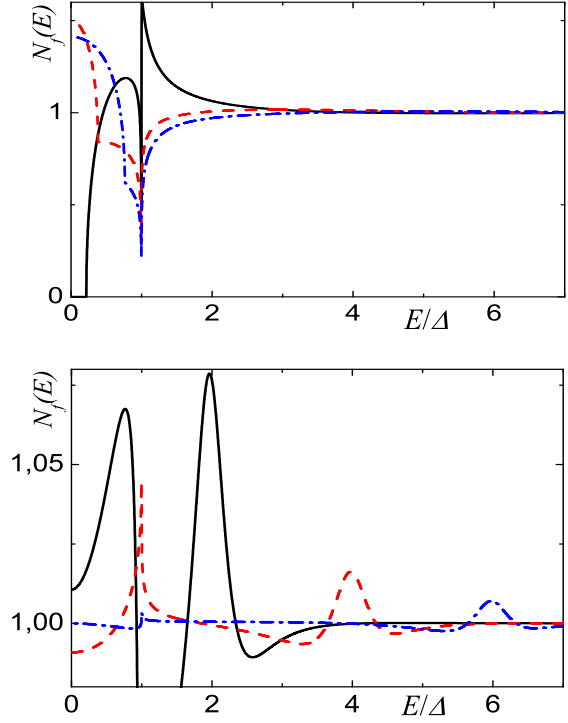


Fig. 3. (Color online) DOS $N_f(E)$ at the outer border of the F layer in the FS bilayer calculated numerically for different values of the exchange field E_{ex} . Parameters of the F/S interface are $\gamma = \gamma_B = 0.01$, $T = 0.1T_c$. Upper panel: $d_f/\xi_n = 1$; lower panel: $d_f/\xi_n = 3$. Solid black line corresponds to $E_{\text{ex}}/\Delta = 2$, dashed red line to $E_{\text{ex}}/\Delta = 4$, and dash-dotted blue line to $E_{\text{ex}}/\Delta = 6$.

At the outer border of the ferromagnet ($x = d_f$) we have $\partial_x \theta_{f\uparrow(\downarrow)} = 0$. At $x = 0$ we use Kupriyanov-Lukichev boundary conditions which in case of the transparent interface is convenient to write as

$$\gamma \partial_x \theta_f|_{x=0} = \partial_x \theta_s|_{x=0}, \quad (9a)$$

$$\xi_n \gamma_B \partial_x \theta_f|_{x=0} = \sinh(\theta_f - \theta_s)|_{x=0}. \quad (9b)$$

Here $\gamma = \sigma_f/\sigma_s$, $\sigma_{f(s)}$ are the conductivities of the F (S) layers correspondingly, $\xi_n = \sqrt{\mathcal{D}/2\pi T_c}$, T_c is the critical temperature of the superconductor, and $\gamma_B = d_f R_T / \xi_n R_F = \xi_s / \xi_n W$. The parameter γ determines the strength of suppression of superconductivity in the S layer near the interface (inverse proximity effect). No suppression occurs for $\gamma = 0$, while strong suppression takes place for $\gamma \gg 1$. In our numerical calculations we assume small $\gamma \ll 1$. Since we consider the transparent interface $R_F \gg R_T$ and contrary to the previous section $W \gg 1$, therefore $\gamma_B \ll 1$. Notice that in the Eqs. (8)-(9) we have omitted the subscripts $\uparrow(\downarrow)$ because equations for both spin directions are identical in the superconductor.

In Fig. 3 we plot the DOS $N_f(E)$ at the outer border of the F layer in the FS bilayer calculated numerically for different values of the exchange field E_{ex} and for different F layer thicknesses d_f . At large enough d_f ($d_f/\xi_n = 3$) we see the peak at $E = E_{\text{ex}}$ [see Fig. 3, lower panel]. For small d_f ($d_f/\xi_n = 1$) the peak is not visible and DOS tends monotonously to unity for $E > \Delta$ [see Fig. 3, upper panel]. The amplitude of the peak is decreasing with increasing E_{ex} : peak is only visible for E_{ex} of the order of few Δ (see [8] for details). We also need to mention that in case of $E_{\text{ex}} < \Delta$ there is no peak in the DOS.

To better illustrate the conditions when the peak at $E = E_{\text{ex}}$ is visible in experiments we consider an analytical limiting case. If the F layer is thick enough ($d_f \gg \xi_f$) and $\gamma = 0$ in Eq. (9), the DOS at the outer border of the ferromagnet can be written as [7,8,43]

$$N_{f\uparrow(\downarrow)}(E) = \text{Re}[\cos \theta_{b\uparrow(\downarrow)}] \approx 1 - \frac{1}{2} \text{Re} \theta_{b\uparrow(\downarrow)}^2. \quad (10)$$

Here $\theta_{b\uparrow(\downarrow)}$ is the boundary value of θ_f at $x = d_f$, given by

$$\theta_{b\uparrow(\downarrow)} = \frac{8F(E)}{\sqrt{F^2(E) + 1} + 1} \exp\left(-p_{\uparrow(\downarrow)} \frac{d_f}{\xi_f}\right), \quad (11)$$

where we use the following notations,

$$p_{\uparrow(\downarrow)} = \sqrt{2/E_{\text{ex}}} \sqrt{-iE_R \pm iE_{\text{ex}}}, \quad (12a)$$

$$F(E) = \frac{\Delta}{-iE_R + \sqrt{\Delta^2 - E_R^2}}, \quad E_R = E + i0. \quad (12b)$$

From Eqs. (10)-(11) we obtain for the full DOS the following expression in the limit $d_f \gg \xi_f$ and for $E \geq \Delta$,

$$N_f(E) = 1 + \sum_{j=\pm} \frac{16\Delta^2 \cos(b_j) \exp(-b_j)}{(E + \varepsilon)(\sqrt{E + \varepsilon} + \sqrt{2\varepsilon})^2}, \quad (13)$$

$$b_j = \frac{2d_f}{\xi_f} \sqrt{\frac{|E + jE_{\text{ex}}|}{E_{\text{ex}}}}, \quad \varepsilon = \sqrt{E^2 - \Delta^2}.$$

We can clearly see the exponential asymptotic of the peak at $E = E_{\text{ex}}$ from Eq. (13). We should keep in mind that Eq. (13) is valid for large d_f/ξ_f , but nevertheless we may qualitatively understand why we do not see the peak at $E = E_{\text{ex}}$ for small ratio of d_f/ξ_f : if this factor is small the variation of the exponent $\exp\{-2(d_f/\xi_f)\sqrt{|E - E_{\text{ex}}|/E_{\text{ex}}}\}$ near the point $E = E_{\text{ex}}$ is also small. The peak is observable only for E_{ex} of the order of a few Δ . For larger exchange fields the peak is difficult to observe, since the energy dependent prefactor of the exponent in Eq. (13) decays as E^{-2} for $E \gg \Delta$.

Detecting this peak one can carefully measure the value of small exchange field $E_{\text{ex}} > \Delta$ in the ferromagnetic metal.

We mention that in the presence of magnetic scattering the DOS peak at $E = E_{\text{ex}}$ do not change the position but gets smeared at large enough scattering rate [8]. We did not consider the effect of domain structure of the F layer on this peak, but we can expect similar behavior as discussed in previous section, i.e. the position of the DOS peak will move to the value of the “effective field” [40].

5. Summary

We propose reliable methods to measure small exchange fields in weak ferromagnet/ superconductor structures. For $E_{\text{ex}} < \Delta$ the subgap differential conductance of the normal metal - ferromagnet - insulator - superconductor (NFIS) junction shows a peak at the voltage bias equal to the exchange field of the ferromagnetic layer, $eV = E_{\text{ex}}$. Thus measuring the subgap conductance one can reliably determine small $E_{\text{ex}} < \Delta$. In the opposite case $E_{\text{ex}} > \Delta$ one can determine the exchange field in scanning tunneling microscopy experiment. The density of states of the FS bilayer measured at the outer border of the ferromagnet shows a peak at the energy equal to the exchange field, $E = E_{\text{ex}}$.

Next we are planning to search for small exchange fields $E_{\text{ex}} > \Delta$ in the experiments, using the ultrahigh vacuum Scanning Tunneling Microscopy (STM) and Spectroscopy (STS) technique, recently developed by one of the authors (Stolyarov et al.) [44].

We also hope that our results will trigger further experimental activity in finding ferromagnetic materials with small exchange fields. Good candidates for such materials can be diluted ferromagnetic alloys (like PdNi, PdFe, CuNi, etc.) and normal metals in proximity with the ferromagnetic insulators (FI). In the latter case the ferromagnetic insulator may induce the exchange field in the thin normal metal layer, placed on top of the FI material.

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